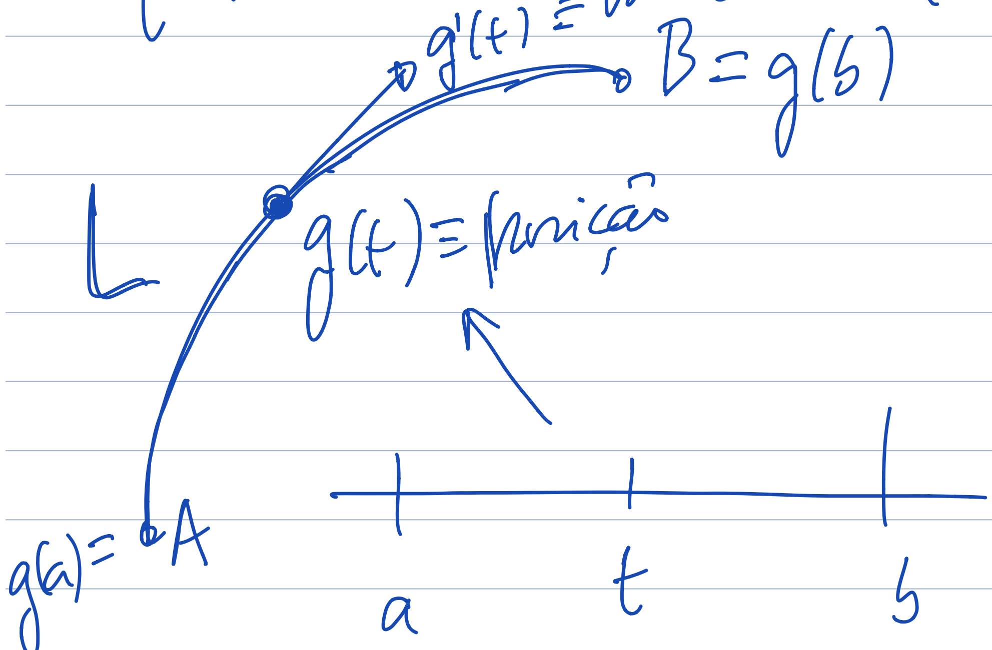
CDI-II — Prática 17/5/21

Ficha 10 + Ficha 11

F.10

Integral de linha de um campo escalar.

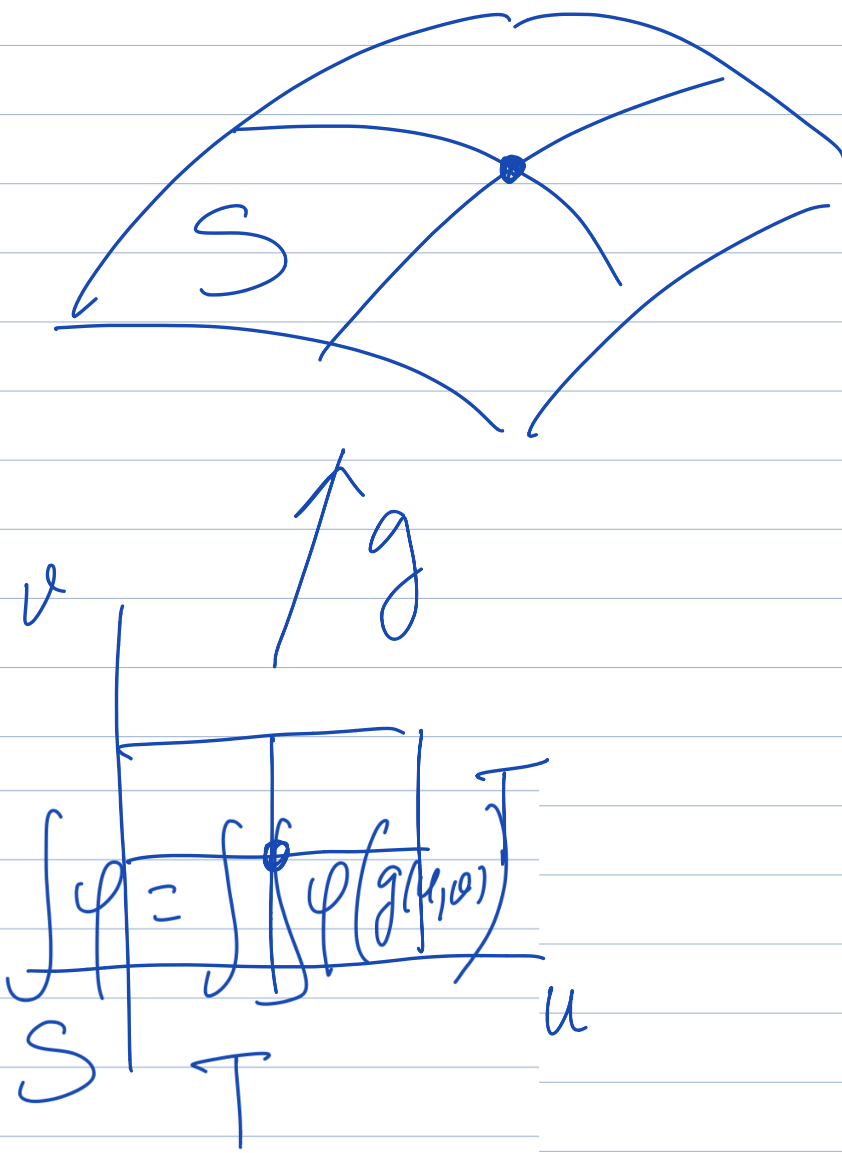
$\varphi : \mathbb{R}^n \to \mathbb{R}$



$$\int_L \varphi = \int_a^b \varphi(g(t))\,\|g'(t)\|\,dt$$

Integral de superficie de un campo escalar.

$$\varphi : \mathbb{R}^n \to \mathbb{R}, \quad S \subset \mathbb{R}^3$$
$$(\dim S = 2)$$

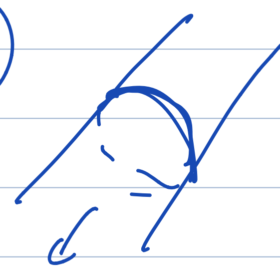


$$\int_S \varphi = \iint_T \varphi(g(u,v))$$

$$\int_S \varphi = \iint_T \varphi(g(u,v)) \sqrt{\det Dg^T Dg}\; du\, dv$$

6 – $L: \begin{cases} x = \underbrace{y^2+z^2}_{\geq 0} \\ x^2+y^2+z^2 = 2 \end{cases} \longrightarrow$ parametriza $\boxed{g(t)}$?

$$\begin{cases} x = y^2+z^2 \\ \underbrace{x^2+x-2=0}_{(x+2)(x-1)=0} \end{cases} \qquad \begin{cases} y^2+z^2=1 \\ x=1 \end{cases}$$



$g(t) = (x(t), y(t), z(t))$

$g(t) = (1, \cos t, \operatorname{sen} t)$, $0<t<2\pi$

$$(\bar{x}, \bar{y}, \bar{z})$$

$$\bar{x} = \frac{\int_L x}{\int_L 1} \qquad \varphi(x,y,z) = x$$

$$\int_L x = \int_0^{2\pi} \varphi(g(t)) \, \|g'(t)\| \, dt$$

$$= \int_0^{2\pi} \|g'(t)\| \, dt \qquad g'(t) = (0, -\sin t, \cos t)$$

$$= \int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t} \, dt = 2\pi \; //$$

etc. $\boxed{\bar{x} = 1}$

$$\bar{y} = \frac{1}{2\pi}\int_L y \qquad \varphi(x,y,z) = y$$

$$\varphi(g(t)) = \cos t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \cos t \, dt = 0 .$$

$$\bar{z} = \frac{1}{2\pi}\int_L z \qquad \varphi(x,y,z) = z$$

$$\varphi(g(t)) = \operatorname{sen} t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \operatorname{sen} t \, dt = 0$$

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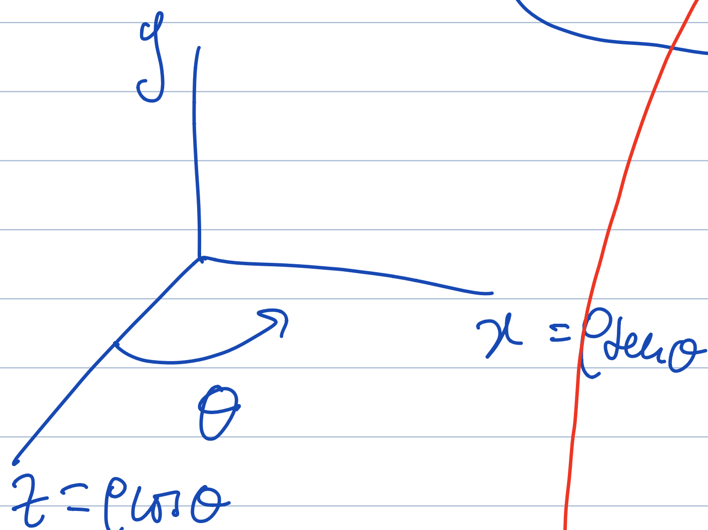
7- a) $A: \quad 1+\underbrace{\sqrt{x^2+z^2}} = y$

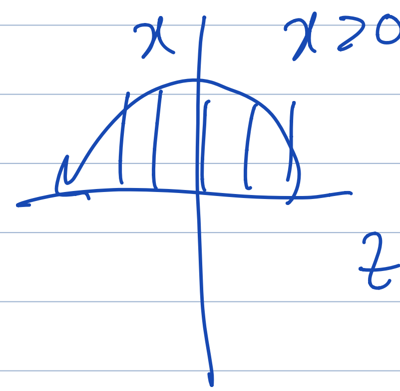
$y<2$
$x>0$

$\dim(A) = 2$

(ρ, θ, y)

$1+\rho = y$





$0 < \theta < \pi$

$$g(\rho,\theta) = (x(\rho,\theta), y(\rho,\theta), z(\rho,\theta))$$

$$g(\rho,\theta) = (\rho \operatorname{sen}\theta, 1+\rho, \rho\cos\theta)$$

$T: \; 0<\rho<1 \; ; \; 0<\theta<\pi$

$$Dg^T Dg = \begin{bmatrix} \|T_1\|^2 & T_1 \cdot T_2 \\ T_2 \cdot T_1 & \|T_2\|^2 \end{bmatrix}_{2\times 2}$$

$$T_1 = D_\rho g = (\operatorname{sen}\theta, 1, \cos\theta)$$

$$T_2 = D_\theta g = (\rho\cos\theta, 0, -\rho\operatorname{sen}\theta)$$

$$\det Dg^T Dg = \det \begin{bmatrix} 2 & 0 \\ 0 & \rho^2 \end{bmatrix} = 2\rho^2$$

$$\sqrt{\det Dg^T Dg} = \sqrt{2\rho^2} = \sqrt{2}\,\rho$$

$$\mathrm{Vol}_2(A) = \int_A 1 = \iint_T \sqrt{\det Dg^T Dg}\, d\rho\, d\theta$$

$$= \int_0^{\pi} \left(\int_0^1 \sqrt{2}\, \rho\, d\rho \right) d\theta$$

$$= \int_0^{\pi} \frac{\sqrt{2}}{2}\, d\theta = \frac{\sqrt{2}}{2}\pi \ //$$

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7-b) B: $z = xy$

$x^2 + y^2 < 1$

Parametrizar B.

$$g(x,y) = (x, y, xy)$$

$$T = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$$

$$D_x g = (1, 0, y)$$

$$D_y g = (0, 1, x)$$

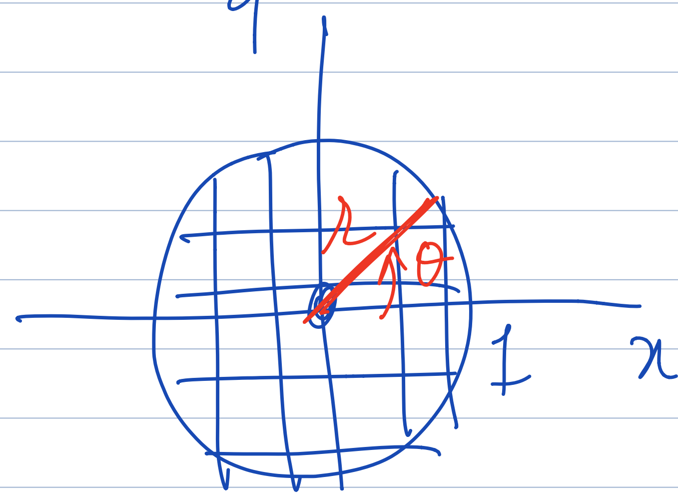
$$\det Dg^T Dg = \det \begin{bmatrix} 1+y^2 & xy \\ xy & 1+x^2 \end{bmatrix}$$

$$= (1+y^2)(1+x^2) - x^2y^2$$

$$= 1 + y^2 + x^2 + x^2y^2 - x^2y^2$$

$$= 1 + x^2 + y^2$$

$$\mathrm{Vol}_2(B) = \iint_T \sqrt{1+x^2+y^2}\, dx\,dy$$



$$\mathrm{Vol}_2(B) = \int_0^{2\pi} \left(\int_0^1 r\sqrt{1+r^2}\, dr \right) d\theta$$

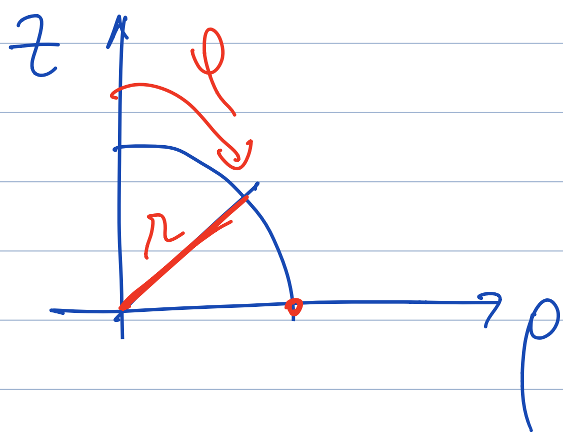
etc . . .

8- $S: x^2+y^2+z^2=a^2$

$(r, \boxed{\theta, \varphi})$ $\qquad \rho^2+z^2=a^2 \qquad z \geq 0$

$\boxed{a>0}$

raio



$\boxed{r=a}$

$$g(\theta,\varphi)=(x(\theta,\varphi), y(\theta,\varphi), z(\theta,\varphi))$$

$$g(\theta,\varphi)=(a\,\mathrm{sen}\,\varphi\cos\theta, a\,\mathrm{sen}\,\varphi\,\mathrm{sen}\,\theta, a\cos\varphi)$$

$$T \left| \begin{array}{l} 0 \leq \theta \leq 2\pi \\ 0 \leq \varphi \leq \frac{\pi}{2} \end{array} \right.$$

etc..

$$\sqrt{\det Dg^T Dg} = a^2\,\mathrm{sen}\,\varphi$$

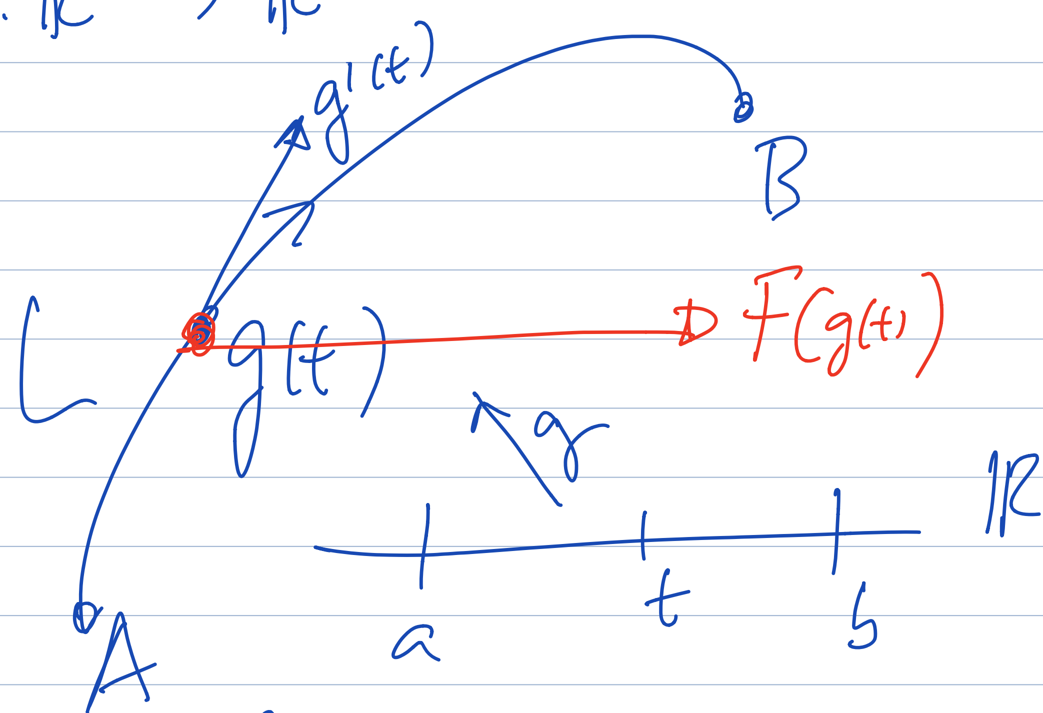
$$\int_S (x^2+y^2) \qquad \varphi(x,y,z) = x^2+y^2$$

$$= \int_0^{2\pi} \left(\int_0^{\frac{\pi}{2}} a^2 \operatorname{sen}\varphi \; a^2 \operatorname{sen}^2\varphi \, d\varphi \right) d\theta$$

$$= a^4 \times 2\pi \int_0^{\frac{\pi}{2}} \operatorname{sen}\varphi \left(1 - \cos^2\varphi\right) d\varphi$$

Ficha 11: Trabalho, TFC.

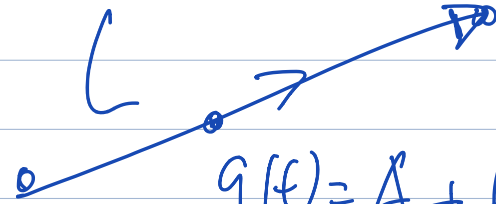
$F: \mathbb{R}^n \to \mathbb{R}^n$



$$W = \int_L F \equiv \int_L F \cdot dg$$

$$W = \int_a^b \underbrace{F(g(t))} \cdot \underbrace{g'(t)}\, \underbrace{dt}$$

2 - a)



$(1,2,3)$ $B = g(1)$

$g(t) = A + t(B-A)$

$g(0) = A$ $(0,0,0)$

$\boxed{0 \le t \le 1}$

$$g'(t) = (B-A)$$

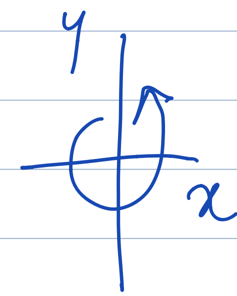
$$g(t) = (t, 2t, 3t) \quad ; \quad g'(t) = (1,2,3)$$

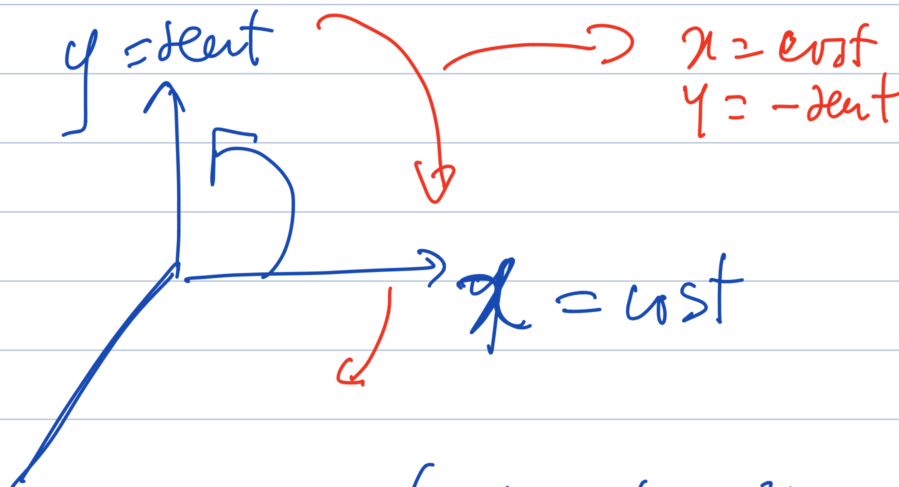
$$W = \int_0^1 f(g(t)) \cdot g'(t)\,dt$$

$$= \int_0^1 (t, 3t, 4t) \cdot (1,2,3)\,dt$$

$$= \int_0^1 (t + 6t + 12t)\,dt = \frac{19}{2} \quad //$$

2-b) $L: \begin{cases} x^2+y^2=1 \\ z = x^2-y^2 \end{cases}$





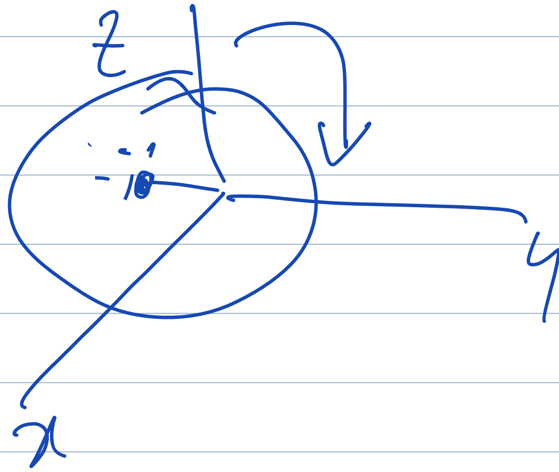
$$g(t) = (\cos t, \operatorname{sen} t, \cos^2 t - \operatorname{sen}^2 t)$$

$$0 < t < 2\pi$$

etc.

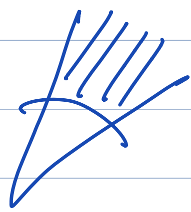
11

2-c) $\begin{cases} x = y^2+z^2 \\ 2y + x = 3 \end{cases}$



$\cos(-t) = \cos t$

$\operatorname{sen}(-t) = -\operatorname{sen} t$



$$\begin{cases} x = y^2 + z^2 \\ x = 3 - 2y \end{cases} \qquad \begin{cases} 3 - 2y = y^2 + z^2 \\ x = 3 - 2y \end{cases}$$

$$\begin{cases} \overbrace{y^2 + 2y + 1}\, + z^2 = 3 + 1 \\ \_\_\_\_\_\_ \end{cases} \qquad \begin{cases} (y+1)^2 + z^2 = 4 \\ x = 3 - 2y \end{cases}$$

$y + 1 = 2\cos t$

$z = -2 \operatorname{sen} t$

$$y = 2\cos t - 1$$

$$z = -2\,\text{sen}\,t$$

$$x = 3 - 2(2\cos t - 1) = 3 - 4\cos t + 2$$

$$= 5 - 4\cos t$$

$$g(t) = \left(5 - 4\cos t,\ 2\cos t - 1,\ -2\,\text{sen}\,t\right)$$

$$0 < t < 2\pi$$

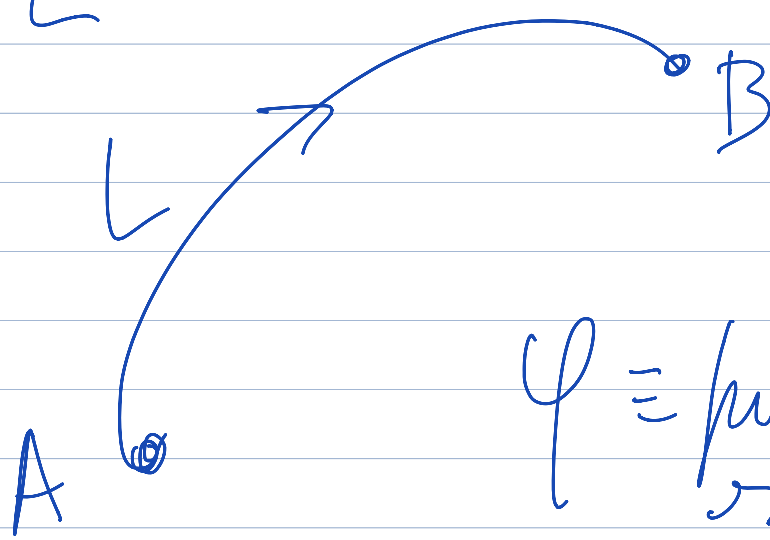
etc.

$F = \nabla \varphi$ $\qquad$ $F: \mathbb{R}^4 \to \mathbb{R}^4$

$\varphi: \mathbb{R}^4 \to \mathbb{R}$

(F é gradiente de φ $\equiv$ F é conservativo).

$$\int_L F = \varphi(B) - \varphi(A) \quad \text{TFC}$$



φ = potencial escalar de F.

Se $\boxed{F = \nabla \varphi}$

$$F_j = \frac{\partial \varphi}{\partial x_j}$$

$j = 1, 2, \ldots, n$

$$\frac{\partial F_j}{\partial x_k} = \frac{\partial}{\partial x_k}\left(\frac{\partial \varphi}{\partial x_j}\right)$$

$$= \frac{\partial}{\partial x_j}\left(\frac{\partial \varphi}{\partial x_k}\right)$$

$$= \frac{\partial F_k}{\partial x_j} \qquad j \neq k$$

então $\boxed{F \text{ é fechado}}$.

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